Chapter one

1. The velocity distribution in a very wide rectangular channel of 3m deep is given by $u=1+2*(\frac{y}{y_o})^{\frac{1}{2}}$, calculate α and β.

Solution

$y_o = 3m$, the river is wide rectangular channel and $u=1+2*\left[\frac{y}{y_o}\right]^{\frac{1}{2}}$

Average velocity $(V) = \frac{1}{A} * \int_A udA$

Total area $A=B*y_o$ and elemental area $dA=B*dy$

$$\text{Thus, } V=\frac{1}{B*y_o} * \int_0^{y_o} \left[1 + 2 * \left[\frac{y}{y_o}\right]^{\frac{1}{2}}\right] B * dy$$

$$=\frac{1}{y_o}\left[y_o + \frac{4}{3} * y_o\right]$$

$$= \frac{7}{3} m/sec$$

Kinetic energy correction factor α

$$\alpha=\frac{1}{A*V^3} * \int_A u^3 \, dA$$

$$\alpha=\frac{1}{B*y_o*\left[\frac{7}{3}\right]^3} * \int_0^{y_o} \left[1 + 2 * \left[\frac{y}{y_o}\right]^{\frac{1}{2}}\right]^3 B * dy \text{ --- (a)}$$

Let $x = 1 + 2 * \left[\frac{y}{y_o}\right]^{\frac{1}{2}}$, then $\sqrt{y} = \left(\frac{x-1}{2}\right) * \sqrt{y_o}$

By deriveting $dx=\frac{dy}{\sqrt{y}*\sqrt{y_o}} \Longrightarrow dy=\sqrt{y} * \sqrt{y_o} * dx$

$$= \left(\frac{x-1}{2}\right) y_o dx$$

Boundaries as y=0, $x=1+2*\left[\frac{0}{y_o}\right]^{\frac{1}{2}}=1$ and as $y=y_o$, $x=1+2*\left[\frac{y_o}{y_o}\right]^{\frac{1}{2}}=3$

Substituting these value into equation (a)

$$\alpha= \frac{1}{y_o*\left[\frac{7}{3}\right]^3} * \int_1^3 x^3 * \left(\frac{x-1}{2}\right) y_o dx$$

$$\alpha = \frac{27}{343} * \left(\frac{\frac{x^5}{5}-\frac{x^4}{4}}{2}\right)\binom{3}{1}]$$

$$\alpha= 1.117784$$

Similarly momentum correction factor β

$$\beta = \frac{1}{A*V^2} * \int_A u^2\, dA$$

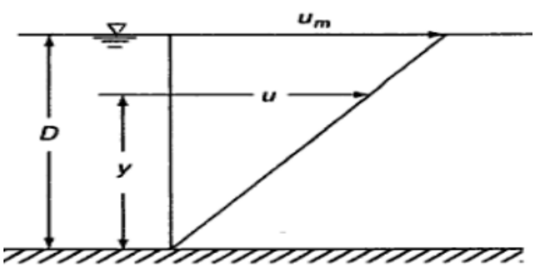
$$\beta = \frac{1}{B*y_o*\left[\frac{7}{3}\right]^2} * \int_0^{y_o} \left[1 + 2*\left[\frac{y}{y_o}\right]^{\frac{1}{2}}\right]^2 B*dy$$

$$\beta = \frac{1}{y_o*\left[\frac{7}{3}\right]^2} * \int_1^3 x^2 * \left(\frac{x-1}{2}\right) y_o dx$$

$$\beta = \frac{9}{49} * \left(\frac{\frac{x^4}{4} - \frac{x^3}{3}}{2}\right) \binom{3}{1}]$$

$$\beta = 1.040816$$

1. Determine the kinetic energy correction factor α and momentum correction factor β for both the velocity profiles.



Solution

From linear relation $\frac{y}{D} = \frac{U}{Um}, U = \frac{y}{D}Um$

$$V = \frac{1}{BD}\int U dA$$

$$V = \frac{1}{BD}\int_0^D \left(\frac{y}{D}Um\right) Bdy$$

$$V = \frac{1}{2}Um$$

Determination of kinetic correction factor α

$$\alpha = \frac{1}{A*v^3}\int U^3 dA$$

$$\alpha = \frac{1}{BD*\left(\frac{1}{2}Um\right)^3}\int_0^D \left(\frac{y}{D}Um\right)^3 Bdy$$

$$\alpha = 2$$

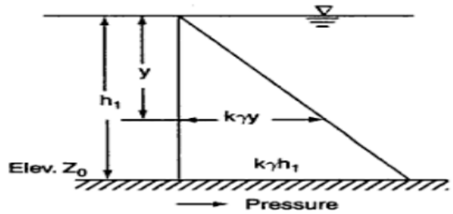
Determination of momentum correction factor

$$\beta = \frac{\int_A u^2 dA}{V^2 A}$$

$$\beta = \frac{1}{BD\left(\frac{1}{2}Um\right)^2}\int_0^D \left(\frac{y}{D}Um\right)^2 Bdy$$

$$\beta = 1.333$$

2. For pressure distribution shown as figure below in an open channel flow, calculate the effective piezometric head. Take the hydrostatic pressure distribution as the reference.



Solution

Zo= elevation of channel bed level above datum.

h_1 = depth of flow

Let h_p= piezometric head at depth y below water surface

Then, $h_p = Z_0 + \frac{P}{\gamma} + (h_1 - y)$, where P=kγy

$$h_p = Z_0 + \frac{k\gamma y}{\gamma} + (h_1 - y)$$

$$h_p = Z_0 + h_1 + (k-1)y$$

Let $\Delta h = (k-1)y$

Effective piezometric head (h_{ep})

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1}\int_0^{h_1} \Delta h dy$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1}\int_0^{h_1} (k-1)y dy$$

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1}(k-1)\frac{y^2}{2}\Big|_0^{h_1}$$

Simplifying this equation

$$\mathbf{h_{ep} = Z_0 + \frac{h_1}{2}(k+1)}$$

***EXAMPLE* 4** The velocity distribution in a rectangular channel of width B and depth of flow y_0 was approximated as $v = k_1\sqrt{y}$ in which k_1 = a constant. Calculate the average velocity for the cross-section and correction coefficients α and β.

Solution

Area of cross-section $A = B y_0$

Average velocity $V = \frac{1}{B y_0}\int_0^{y_0} v(B\,dy)$

$$= \frac{1}{y_0}\int_0^{y_0} k_1\sqrt{y}dy = \frac{2}{3}k_1\sqrt{y_0}$$

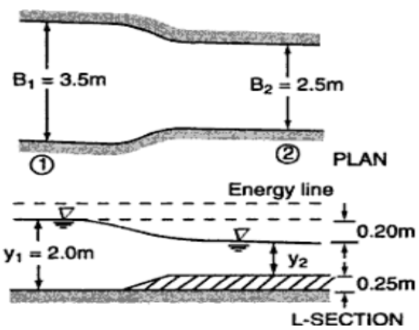
Kinetic energy correction factor α $= \frac{\int_0^{y_0} v^3(B\,dy)}{V^3 B y_0} = \frac{\int_0^{y_0} k_1^3\ y^{3/2}\ B\ dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^3 B y_0} = 1.35$

Momentum correction factor β $= \frac{\int_0^{y_0} v^2(B\,dy)}{V^2 B y_0} = \frac{\int_0^{y_0} k_1^2 y B\,dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^2 B y_0} = 1.125$

Chapter two

Example 13. The width of a horizontal rectangular channel is reduced from 3.50m to 2.50m and the floor is raised by 0.25m in elevation at a given section. At the upstream section, the depth of flow is 2.00m and the kinetic energy correction factor α is 1.15 if the drop in the water surface elevation is 0.20m and the kinetic energy correction factor at contracted section α is unity, calculate the discharge if

a) Energy loss is neglected
b) The energy loss is one-tenth of the upstream velocity head



Solution

Referring to Fig. above $y_1 = 2.00$m

$$y_2 = 2.0 - 0.25 - 0.20 = 1.55 \text{ m}$$

By continuity

$$B_1 y_1 V_1 = B_2 y_2 V_2$$

$$V_1 = \frac{2.5 \times 1.55}{3.5 \times 2.0} V_2 = 0.5536 V_2$$

(a) *When there is no energy loss*
By energy equation applied to Sections 1 and 2,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

$$\alpha_1 = 1.15 \text{ and } \alpha_2 = 1.0$$

$$\frac{V_2^2 - (1.15 V_1^2)}{2g} = y_1 - y_2 - \Delta Z$$

$$\frac{V_2^2}{2g}\left[1 - (1.15)(0.5536)^2\right] = 2.00 - 1.55 - 0.25$$

$$0.6476 \frac{V_2^2}{2 \times 9.81} = 0.2$$

$$V_2 = 2.462 \text{ m/s}$$

Discharge $Q = 2.5 \times 1.55 \times 2.462 = 9.54 \text{ m}^3/\text{s}$

(a) ***When there is no energy loss***

By energy equation applied to Sections 1 and 2,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

$$\alpha_1 = 1.15 \text{ and } \alpha_2 = 1.0$$

$$\frac{V_2^2 - (1.15V_1^2)}{2g} = y_1 - y_2 - \Delta Z$$

$$\frac{V_2^2}{2g}\left[1-(1.15)(0.5536)^2\right] = 2.00 - 1.55 - 0.25$$

$$0.6476 \frac{V_2^2}{2 \times 9.81} = 0.2$$

$$V_2 = 2.462 \text{ m/s}$$

Discharge $Q = 2.5 \times 1.55 \times 2.462 = 9.54 \text{ m}^3/\text{s}$

(b) ***When there is an energy loss***

$$H_L = 0.1\left[\alpha_1 \frac{V_1^2}{2g}\right] = 0.115 \frac{V_1^2}{2g}$$

By energy equation,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g} + H_L$$

$$\left[\alpha_2 \frac{V_2^2}{2g} - \alpha_1 \frac{V_1^2}{2g} + H_L\right] = y_1 - y_2 - \Delta Z$$

Substituting $\alpha_2 = 1.0$, $\alpha_1 = 1.15$ **and** $H_L = 0.115 \frac{V_1^2}{2g}$

$$\frac{V_2^2}{2g} - 1.15\frac{V_1^2}{2g} - 0.115\frac{V_1^2}{2g} = 2.00 - 1.55 - 0.25$$

Since $V_1 = 0.5536 V_2$

$$\frac{V_2^2}{2g}\left[1-(0.9)(1.15)(0.5536)^2\right] = 0.2$$

$$\frac{0.6826\, V_2^2}{2 \times 9.81} = 0.2$$

$V_2 = 2.397$ **m/s and discharge** $Q = 2.5 \times 1.55 \times 2.397 = 9.289 \text{ m}^3/\text{s}$

<u>Example 2 :</u> A rectangular channel 3.00m wide carries a discharge of 10.00m3/sec. and has its specific energy of 2.00m water. Calculate alternate depths and corresponding Froude numbers.

<u>Solution:</u>

$$E_s = y + \frac{V^2}{2g} = y + \frac{Q^2}{2g\ y^2 B^2}$$

$$2 = y + \frac{10^2}{2*9.81*3^2 y^2}$$

Solving by trial and error y_1 =1.84m and y_2 =0.64m

$$Fr_1 = \frac{V}{\sqrt{gy_1}} = \frac{10}{3*1.84*\sqrt{9.81*1.84}} = 0.4264 < 1$$

Since $Fr_1 < 1$the flow is subcritical

$$Fr_2 = \frac{V}{\sqrt{gy_2}} = \frac{10}{3*0.64*\sqrt{9.81*0.64}} = 2.5056 > 1$$

Since $Fr_1 > 1$the flow is supercritical

EXAMPLE 2. **Calculate the critical depth and the corresponding specific energy for a discharge of 5.0 m^3/s in the following channels:**

(a) Rectangular channel, B = 2.0 m
(b) Triangular channel, m = 0.5
(c) Trapezoidal channel, B = 2.0 m, m = 1.5
(d) Circular channel, D = 2.0 m

Solution

(a) Rectangular Channel

$$q = Q/B = \frac{5.0}{2.0} = 2.5 \text{ m}^3/\text{s/m}$$

$$y_c = (q^2/g)^{1/3} = \left[\frac{(2.5)^2}{9.81}\right]^{1/3} = 0.860 \text{ m}$$

Since for a rectangular channel $\frac{E_c}{y_c} = 1.5$, $\quad E_c = 1.290$ m

(b) *Triangular Channel*

From Eq. (2.14) $\quad y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$

$$= \left[\frac{2 \times (5)^2}{9.81 \times (0.5)^2}\right]^{1/5} = 1.828 \text{ m}$$

Since for a triangular channel $\frac{E_c}{y_c} = 1.25$, $E_c = 2.284$ m

(c) *Trapezoidal Channel*

$$\psi = \frac{Qm^{3/2}}{\sqrt{g}\,B^{5/2}} = \frac{5.0 \times (1.5)^{3/2}}{\sqrt{9.81} \times (2.0)^{5/2}} = 0.51843$$

Using Table 2A.2 the corresponding value of

Using Table 2A.2 the corresponding value of

$$\zeta_c = \frac{my_c}{B} = 0.536$$

$$y_c = 0.715 \text{ m}$$
$$A_c = (2.0 + 1.5 \times 0.715) \times 0.715 = 2.197 \text{ m}^2$$
$$V_c = 5.0/2.197 = 2.276 \text{ m/s}$$
$$V_c^2/2g = 0.265 \text{ m}$$

$$E_c = y_c + \frac{V_c^2}{2g} = 0.715 + 0.264 = 0.979 \text{ m}$$

(d) *Circular Channel*

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{5.0}{\sqrt{9.81}} = 1.5964$$

$$Z_c/D^{2.5} = 0.2822.$$

From Table 2A.1 showing $Z/D^{2.5}$ vs y/D, the corresponding value of y_c/D by suitable linear interpolation is

$$\frac{y_c}{D} = 0.537, \; y_c = 1.074 \text{ m}$$

Also, from Table 2A.1, for $\frac{y_c}{D} = 0.537, \frac{A_c}{D^2} = 0.4297$

Hence
$$A_c = (2.0)^2 + 0.4297 = 1.7187 \text{ m}^2$$
$$V_c = 5.0/1.7187 = 2.909 \text{ m/s}$$
$$V_c^2/2g = 0.264 \text{ m}$$

$$E_c = y_c + \frac{V_c^2}{2g} = 1.074 + 0.431 = 1.505 \text{ m}$$

***EXAMPLE* 3** Calculate the bottom width of a channel required to carry a discharge of 15.0 m^3/s as a critical flow at a depth of 1.2 m, if the channel section is (a) rectangular and (b) trapezoidal with side slope of 1.5 horizontal : 1 vertical.

Solution

(a) *Rectangular Section*

The solution here is straightforward

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \quad \text{i.e.} \quad q = \sqrt{g y_c^3}$$

$$q = \sqrt{9.81(1.2)^3} = 4.117 \text{ m}^3/\text{s}/\text{m}$$

$$B = \text{bottom width} = \frac{15.0}{4.117} = 3.643 \text{ m}$$

(b) *Trapezoidal Channel*

The solution in this case is by trial-and-error.

$$A_c = (B + 1.5 \times 1.2) \times 1.2 = (B + 1.8) \times 1.2$$
$$T_c = (B + 2 \times 1.5 \times 1.2) = (B + 3.6)$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$\frac{(B + 1.8)^3 \times (1.2)^3}{(B + 3.6)} = \frac{(15)^2}{9.81}$$

$$\frac{(B + 1.8)^3}{(B + 3.6)} = 13.273$$

By trial-and-error $\quad B = 2.535$ m

EXAMPLE 4 **Find the critical depth for a specific energy head of 1.5 m in the following channels:**

(a) Rectangular channel, $B = 2.0$ m
(b) Triangular channel, $m = 1.5$
(c) Trapezoidal channel, $B = 2.0$ m and $m = 1.0$
(d) Circular channel, $D = 1.50$ m

Solution

(a) *Rectangular Channel*

By Eq. (2.10) $\quad E_c = \frac{3}{2} y_c = 1.50\text{m}$

$$y_c = \frac{1.50 \times 2}{3} = 1.00\text{m}$$

(b) *Triangular Channel*

By Eq. (2.15) $\quad E_c = 1.25\, y_c = 1.50$ m

$$y_c = \frac{1.50}{1.25} = 1.20\text{m}$$

(c) ***Trapezoidal Channel***

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{Q^2}{2g A_c^2}$$

Since by Eq. (2.4a) $\quad \frac{Q^2}{g} = A_c^3 / T_c,\ E_c = y_c + \frac{A_c}{2T_c}$

$$1.5 = y_c + \frac{(2.0 + y_c)y_c}{2(2.0 + 2y_c)}$$

Solving by trial and error, $y_c = 1.095$ m.

(d) ***Circular Channel***

$$E_c = y_c + \frac{A_c}{2T_c}$$

By non-dimensionalising with respect to the diameter D.

$$\frac{y_c}{D} + \frac{(A_c / D^2)}{2(T_c / D)} = \frac{E_c}{D} = \frac{1.5}{1.5} = 1.0$$

From Table 2A.1, values of (A_c/D^2) and (T_c/D) for a chosen (y_c/D) are read and a trial and error procedure is adopted to solve for y_c/D. It is found that

$$\frac{y_c}{D} = 0.69 \quad \text{and} \quad y_c = 0.69 \times 1.50 = 1.035\,\text{m}$$

Example 5 Show that the critical depth y_c is related to alternate depths y_1 and y_2 in rectangular channel by the equation:

$$y_c = \left(\frac{2y_1^2 y_2^2}{y_1 + y_2}\right)^{\frac{1}{3}}$$

Solution: At alternate depths y_1 and y_2, specific energy is same, i.e., $E_1 = E_2$.

$\therefore$
$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

or
$$y_1 + \frac{1}{2g}\left(\frac{Q^2}{B^2}\right)\frac{1}{y_1^2} = y_2 + \frac{1}{2g}\left(\frac{Q^2}{B^2}\right)\frac{1}{y_2^2}$$

or
$$y_1 + \left(\frac{q^2}{g}\right)\frac{1}{2y_1^2} = y_2 + \left(\frac{q^2}{g}\right)\frac{1}{y_2^2} \qquad \because \frac{Q}{B} = q$$

or
$$y_1 + \frac{y_c^3}{2y_1^2} = y_2 + \frac{y_c^3}{y_2^2} \qquad \because \left(\frac{q^2}{g}\right) = y_c^3$$

Multiplying by $2y_1^2 y_2^2$,

$$2y_1^3 y_2^2 + y_2^2 y_c^3 = 2y_1^2 y_2^3 + y_1^2 y_c^3$$

or
$$y_2^2 y_c^3 - y_1^2 y_c^3 = 2y_1^2 y_2^3 - 2y_1^3 y_2^2$$
$$y_c^3 (y_2^2 - y_1^2) = 2y_1^2 y_2^2 (y_2 - y_1)$$
$$y_c^3 = \frac{2y_1^2 y_2^2 (y_2 - y_1)}{(y_2 + y_1)(y_2 - y_1)}$$

Thus,
$$y_c = \left(\frac{2y_1 y_2}{y_2 + y_1}\right)^{1/3}$$

Example 6 In a rectangular channel F_1 and F_2 are the Froude numbers corresponding to the alternate depths of a certain discharge. Show that

$$\left(\frac{F_2}{F_1}\right)^{\frac{2}{3}} = \frac{2+F_2^2}{2+F_1^2}$$

Solution:

F_1 and F_2 are Froude numbers of alternate depths y_1 and y_2 respectively

Since y_1 and y_2 are alternate depths they have the same specific energy

Thus, $E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g}$

$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$, divide both side by $y_2 y_1$

$$\frac{1}{y_2} + \frac{V_1^2}{2gy_2y_1} = \frac{1}{y_1} + \frac{V_2^2}{2gy_2y_1}$$

$\frac{1}{y_2}\left[1 + \frac{V_1^2}{2gy_1}\right] = \frac{1}{y_1}\left[1 + \frac{V_2^2}{2gy_2y_1}\right]$, where $F_1^2 = \frac{V_1^2}{gy_1}$ and $F_2^2 = \frac{V_2^2}{2gy_2y_1}$

$\frac{y_1}{y_2} = \left[\frac{2+F_2^2}{2+F_1^2}\right]$, but $F_1^2 = \frac{q^2}{gy_1^3}$ and $F_2^2 = \frac{q^2}{gy_2^3}$ subsisting these value and simplifying in the equation

$$\left(\frac{F_2}{F_1}\right)^{\frac{2}{3}} = \frac{2+F_2^2}{2+F_1^2}$$

Example 7:- A rectangular channel has a width of 2.0 m and carries a discharge 4.80 m3/sec with a depth of 1.60 m. At a certain cross-section a small, smooth hump with a flat top and a height 0.10m is proposed to be built. Neglect the energy loss.

A). Calculate the likely change in the water surface.

B). In this example if the height of the hump is 0.50 m, estimate the water surface elevation on the hump and at a section upstream of the hump.

Solution:

A), Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively

$q = 4.8/2 = 2.4 m^3/se/m$

$v_1 = 2.4/1.6 = 1.5 m/se$, $\frac{V_1^2}{2g} = \frac{1.5^2}{2*9.81} = 0.115m$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{1.5}{\sqrt{9.81*1.6}} = 0.38$$

The upstream flow is subcritical and the hump will cause a drop in the water surface elevation. The specific energy at section 1 is,

$E_1 = 1.6 + 0.115 = 1.715m$

$E_2 = E_1 - \Delta Z = 1.715 - 0.1 = 1.615m$

$$y_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} = \left[\frac{2.4^2}{9.81}\right]^{\frac{1}{3}} = 0.837m$$

$E_c = 1.5y_c = 1.5*0.837 = 1.26m$

The minimum specific energy at section 2 is Ec2 = 1.26 m < E2 = 1.615 m. Hence $y_2 > y_c$ and the upstream depth y_1 will remain unchanged. The depth y_2 is calculated by solving the specific energy equation,

$$E_2 = y_2 + \frac{V_2^2}{2g} \quad , \qquad 1.615 = y_2 + \frac{2.4^2}{2*9.81*y_2^{\,2}}$$

Solving by trial and error gives, y_2= 1.48 m.

The drop at water surface elevation is,

$\Delta y = 1.60 - 1.48 - 0.10 = 0.02m$

B), Available energy at section 2 is,

$E_2 = E_1 - \Delta Z = 1.715 - 0.5 = 1.215m$

$E_{c2} = 1.5*y_{c2} = 1.5*0.837 = 1.26m$

The minimum specific energy required at section 2 is greater than E2, (Ec2 = 1.26 m =1.215 m), the available specific energy at that section. Hence, the depth at section 2 be at the critical depth. Thus E2 = Ec2 = 1.26 m. The upstream depth y_1 will increase depth y_1' such that the new specific energy at the upstream section 1 is,

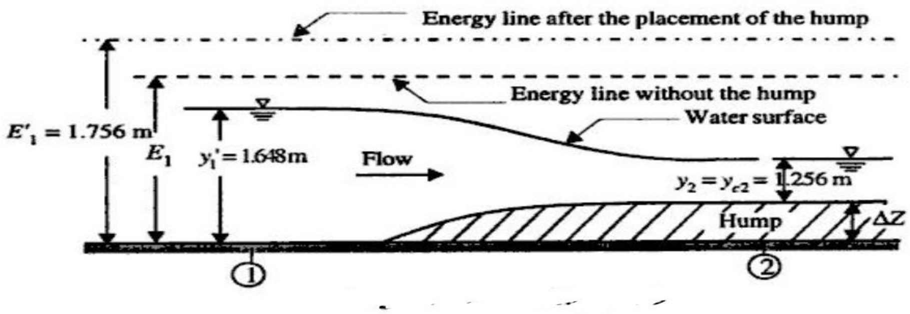
$$E_1' = E_{c2} + \Delta Z$$

$$y_1' + \frac{V_1'}{2g} = y_1' + \frac{q^2}{2gy_1'} = E_{c2} + \Delta Z = 1.26 + 0.5 = 1.76m$$

$$y_1' + \frac{2.4^2}{2*9.81*y_1'} = 1.76m$$

Solving by trial and error and selecting the positive root gives, $y'_1 > y_2$, y'_1 = 1.648 m.

The nature of the water surface is shown in Fig.



Example 8:-A rectangular channel with a discharge 25m3/sec. bottom width of 6.25m, depth y=2m is contracted to 5.75m.

I. Find the depth at contraction and width at contraction.

II. When the depth at contraction is critical, what will be the width at contraction?

Solution:

$$I).E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{25^2}{2*9.81*(6.25*2)^2} = 2.20387m$$

Assuming no loss in contraction, let y_2 is the depth at contraction. Then

$$E_1 = E \text{ at contraction} = 2.20387 = y_2 + \frac{25^2}{2*9.81*(5.75*y_2^2)}$$

Solving by trial and error y_2 =1.936m

II. Let width at contraction is B_c, assuming no loss,

$$E_1 = E_{c=} y_c + \frac{Q^2}{A_c^2 2g}$$

In rectangular channel,

$$E_{c=} \frac{3}{2} y_c, y_c = \frac{2}{3} E_c = \frac{2}{3} * 2.20387$$

y_c =1.4692m

$$E_{c=} y_c + \frac{Q^2}{A_c^2 2g}, \qquad 2.20387 = 1.4692 + \frac{25}{B_c^2 y_c^2 * 2 * g}$$

$$0.7346 = \frac{625}{B_c^2 * 1.4692^2 * 2 * 9.81}$$

B_c =4.482m

Examle 9 A 5.00m wide rectangular channel carries 20m3/sec. of discharge at a depth of 2.00m. The width beyond a certain section is to be changed to 3.50m. If it is desired to keep the water surface elevation unaffected by this change, what modifications are needed to the bottom elevation?

Solution:

Flow Area at upstream $A_1 = B_1 y_1 = 5 * 2 = 10m^2$

Flow velocity at upstream $V_1 = \frac{Q}{A} = \frac{20}{10} = 2m/sec.$

Froude number at upstream $F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{2}{\sqrt{9.81*2}} = 0.005 < 1$

Since Froude number at upstream is less than unity the flow is at subcritical.

$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{2^2}{2*9.81} = 2.204$m of water

Discharge intensity at contracted channel section $q_2 = \frac{Q}{B_2} = \frac{20}{3.5} = 5.714 m^3/sec/m$

Critical depth at contracted section $y_{c2} = \left[\frac{q_2^2}{g}\right]^{\frac{1}{3}} = \left[\frac{5.714^2}{9.81}\right]^{\frac{1}{3}} = 1.493m$

Critical energy at contracted section $E_{c2} = 1.5\, y_{c2} = 1.5 * 1.493 = 2.24m\ of\ water$

If water level is not changed at upstream the specific energy at contracted section should have to be maintained at critical energy at that section, thus $E_{c2} = E_2$

Then, $\Delta Z = E_1 - E_2 = 2.204 - 2.24 = -0.036$m, therefore the channel bed level lowered by 3.6cm.

Example 10. A rectangular channel is 2.50m wide and conveys a discharge of $2.75 m^3/sec.$ at a depth of 0.90m. a contraction of width is proposed at a section in this channel. Calculate the water surface elevations in the contracted section as well as in an upstream 2.50m wide section when the width of the proposed contraction is

a) 2.00m b) 1.50m. (Neglect energy losses in the transition).

Solution

Flow velocity at upstream $V = \frac{Q}{A_1} = \frac{2.75}{2.5*0.9} = 1.22m/sec.$

Froude Number at upstream $F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{1.22}{\sqrt{9.81*0.9}} = 0.41059 < 1$

Since $F_{r1} = 0.41059 < 1$ the flow is at subcritical state at upstream.

Specific energy at upstream $E_1 = y_1 + \frac{V_1^2}{2g} = 0.9 + \frac{1.22^2}{19.62} = 0.976$m of water

Let B2min. is a minimum width that doesn't cause water surface change at upstream

Therefore, $E_{c2}= E_2= E_1$

Then critical depth for rectangular channel, $y_{c2} = \frac{2}{3} * E_1 = \frac{2}{3} * 0.976 = 0.651m$

Discharge intensity can be found from the relation $y_{c2} = \left[\frac{q_2^2}{g}\right]^{\frac{1}{3}}$, hence

$$q_2 = \sqrt{g * y_{c2}^3} = \sqrt{9.81 * 0.651^3} = 1.645\text{m3/sec./m}$$

Again from the relation $B_{2min.} = \frac{Q}{q_2} = \frac{2.75}{1.645} = 1.67m$

a) When B_2= 2.00m, since $B_2 > B_{2min.}$ water surface at upstream remain (i.e $y_1 = 0.9m$) unchanged and since energy losses in the transitions are neglected $E_2= E_1$

Hence, $E_2 = y_2 + \frac{V_2^2}{2g}$, however, $V_2 = \frac{Q}{A_2}$

$$E_2 = y_2 + \frac{Q^2}{2gy_2^2B_2^2}$$

$$0.976 = y_2 + \frac{2.75^2}{2 * 9.81 * y_2^2 * 2^2}$$

Solving by trial and error, $y_2 = 0.839m$

b) When B_2=1.5m, since $B_2 < B_{2min.}$ water surface at upstream should have to increase to y_1' to increase the upstream energy E_1'

$$q_2 = \frac{Q}{B_2} = \frac{2.75}{1.5} = 1.83m^3/sec./m$$

Critical depth at downstream, $y_{c2} = \left[\frac{q_2^2}{g}\right]^{\frac{1}{3}} = \left[\frac{1.83^2}{9.81}\right]^{\frac{1}{3}} = 0.6997m$

Specific energy at downstream is equal to the critical energy at downstream, thus, $E_{c2}= E_{2.}$

$E_{c2} = \frac{3}{2} * y_{c2} = \frac{3}{2} * 0.6997 = 1.0495$m of water

Since energy losses in the transition is neglected $Ec_2= E_2= E_1'$

Therefore,

$$E_1' = y_1' + \frac{Q^2}{2g(y_1')^2B_2^2}$$

$$1.0495\text{m} = y_1' + \frac{2.75^2}{2*9.81*(y_1')^2*2.5^2}$$

Solving by trial and error method, $y_1' = 0.986m$ and $y_2=y_{c2}$=0.6997m

Example 11. A 3.00m wide horizontal rectangular channel is narrowed to a width of 1.50m to cause critical flow in the contracted section. If the depth in contracted section is 0.80m, calculate the discharge in the channel and the possible depths of flow and corresponding Froude numbers in the 3.00m wide section. Neglected energy losses in the transition.

Solution:-

The flow in the contracted section is at critical flow, therefore $y_{c2} = y_2 = 0.80m$

Thus, $y_{c2} = \left[\frac{q_2^2}{g}\right]^{\frac{1}{3}}$

$$q_2 = [y_{c2}^3 * g]^{\frac{1}{2}} = [0.80^3 * 9.81]^{\frac{1}{2}} = 2.24m^3/sec./m$$

Discharge in the channel, $Q = q_2 * B_2 = 2.24 * 1.5 = 3.36m^3/sec.$

Since energy losses in transition are neglected $E_1 = E_2 = E_{C2}$

$E_1 = E_2 = E_{C2} = 1.5 * y_{c2} = 1.5 * 0.80 = 1.20$m of water

Applying specific energy equation at wide channel section

$$E_1 = y_1 + \frac{Q^2}{2gB_1^2 y_1^2}$$

$$1.20 = y_1 + \frac{3.36^2}{19.62 * 3^2 * y_1^2}$$

Solving by trial and error, $y_1 = 1.153m$

Flow velocity at wide channel section, $V_1 = \frac{Q}{B_1 y_1} = \frac{3.36}{3*1.153} = 0.97m/sec.$

Froude number at wide channel, $F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{0.97}{\sqrt{9.81*1.153}} = 0.288418$

Example 12. Water flows at a velocity of 1.00m/sec. and a depth of 2.00m in an open channel of rectangular cross section and bed-width of 3.00m. At certain section the width is reduced to 1.80m and the bed is raised by 0.65m. Will the upstream depth affected and if so, to what extent?

Solution:-

Flow discharge Q = V*A =1*2*3 =6m³/sec.

Froude number at upstream $F_{r1} = \frac{V_1}{\sqrt{g y_1}} = \frac{1}{\sqrt{9.81*2}} = 0.226 < 1$, thus the flow is subcritical

Applying specific energy equation $E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{1}{19.62} = 2.053m\ of\ water$

Assuming energy losses in transitions are neglected,

$E_2 = E_1 - \Delta Z = 2.053 - 0.65 = 1.401m$

Let is critical at contracted section, then for rectangular channel

$y_c = \frac{2}{3}E_c = \frac{2}{3} * 1.401$ =0.934m

Again for rectangular channel, $y_c = \left(\frac{q_{2max}^2}{g}\right)^{\frac{1}{3}}$

$$q_{2max.} = (y_c^3 g)^{\frac{1}{2}} = \sqrt{0.934^3 * 9.81} = 2.827 m^3/sec./m$$

Minimum width at downstream $B_{2min} = \frac{Q}{q_{2max}} = \frac{6}{2.827} = 2.12m > 1.80m$

Since $B_{2min} > B_2$ available energy at section is less than minimum energy at the same section, therefore the available energy is maintained at minimum energy and the upstream depth should have to increased to y_1' and the upstream energy will increased to E_1'

Minimum energy at section2 $E_2 = E_c = 1.5 * \left(\frac{\left(\frac{6}{1.8}\right)^2}{9.81}\right)^{\frac{1}{3}}$ =1.563582m

Since energy losses in transition are assumed to be neglected the increased specific at upstream is

$E_1' = E_2 + \Delta Z = 1.56 + 0.65 = 2.21m$ Of water

Applying specific energy at upstream, $E_1' = y_1' + \frac{Q^2}{2gB_1^2(y_1')^2}$

$$2.21 = y_1' + \frac{6^2}{19.62 * 3^2 * (y_1')^2}$$

Solving by trial and error, $y_1' = 2.17m$

The depth is increased by 2.17-2.00 = 17cm

example 13 Figure 1.30 shows a submerged flow over a sharp-crested weir in a rectangular channel. If the discharge per unit width is $1.8m^3/sec./m$, estimate the energy loss due to the weir. What is the force on the weir plate?

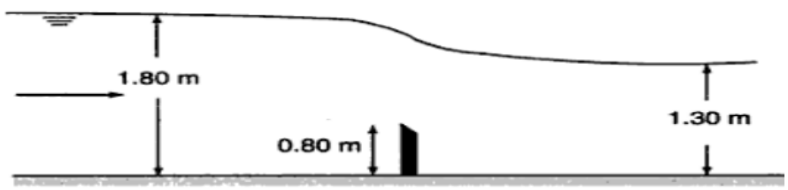


Fig. 1.30 *Problem 1.28*

Solution:

Specific energy at upstream of weir $(E_1) = y_1 + \frac{V_1^2}{2g}$, but where $V_1^2 = \frac{q^2}{y_1^2}$

$$(E_1) = y_1 + \frac{q^2}{2gy_1^2}$$

$$(E_1) = 1.80 + \frac{1.8^2}{2*9.81*1.8^2}$$

$$= 1.851m$$

Specific energy at downstream of weir $(E_2) = y_2 + \frac{V_2^2}{2g}$, but where $V_2^2 = \frac{q^2}{y_2^2}$

$$(E_2) = y_2 + \frac{q^2}{2gy_2^2}$$

$$(E_2) = 1.30 + \frac{1.8^2}{2*9.81*13^2}$$

$$= 1.398m$$

Energy losses due to flow over the weir (ΔE) = (E_1) - (E_2) =1.851-1.398=**0.453m**

Force on the weir plate can be found by applying momentum principle as follow

$\sum F = F_1 - F_2 + w \sin\theta - F_D = \rho Q(\beta_2 v_2 - \beta_1 v_1)$, since the channel is horizontal $\Theta=0$, hence $w \sin\theta=0$

$F_1 - F_2 - F_D = \rho Q(\beta_2 v_2 - \beta_1 v_1)$, where $F_1=\gamma * B*\frac{y_1^2}{2}$, $F_2=\gamma * B*\frac{y_2^2}{2}$, $V_2=\frac{q}{y_2}$ and $V_2=\frac{q}{y_2}$ there fore

$\gamma * B*\frac{y_1^2}{2} - \gamma * B*\frac{y_2^2}{2} - F_D = \rho Q(\beta_2\frac{q}{y_2} - \beta_1\frac{q}{y_2})$, assume $\beta_2=\beta_1=1.00$ and dividing both side by $\gamma * B$

$$\frac{y_1^2}{2} - \frac{y_2^2}{2} - \frac{FD}{\gamma * B} = \frac{q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_2}\right)$$

$$\frac{FD}{\gamma * B} = \frac{y_1^2}{2} - \frac{y_2^2}{2} - \frac{q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_2}\right)$$

$$\frac{FD}{\gamma * B} = \frac{1.8^2}{2} - \frac{1.3^2}{2} - \frac{1.8^2}{9.81}\left(\frac{1}{1.3} - \frac{1}{1.8}\right)$$

$$\frac{FD}{\gamma * B} = 0.704m^2$$

$$\frac{FD}{B} = 0.704m^2 * 9.81KN/m^2 = 6.91KN/m$$

Chapter three

Example 1:- A rectangular channel 3.60m wide had badly-damaged surfaces and had a Manning's roughness coefficient (n=0.03). As a first phase of repair, its bed was lined with concrete (n=0.015). If depth of flow remains same at 1.20m before and after the repair, what is the increase of discharge obtained as result of repair?

Solution

- Wetted perimeter of channel section (P) =B+2y = 3.6+2*1.2 = 6.00m
- Wetted Area (A) = By =3.6*1.2 =4.32m^2
- Hydraulic Radius (R) = $R = \frac{A}{P} = \frac{4.32}{6.00} = 0.72m$
- Discharge (Q_1)= $\frac{AR^{2/3}\sqrt{S}}{n_1}$ --------------------------------a
- During repairs :-
  - ❖ Only the channel bed is lined with concrete (n_r=0.015) and wetted perimeter (P_1=3.60m)
  - ❖ The sides are remain unlined (n_1=0.03) and wetted perimeter of the sides can be (P_2= 2y = 2*1.2 =2.4m)
- Equivalent roughness (n_2) = $\left[\frac{\Sigma\left(n_i^{3/2}P_i\right)}{P}\right]^{2/3} = \left[\frac{0.03^{3/2}*2.4+0.015^{3/2}*3.6}{6.00}\right]^{2/3} = 0.02163$
- Discharge (Q_2)= $\frac{AR^{2/3}\sqrt{S}}{n_2}$ -------------------------------b
- Increase in discharge can be determined by dividing equation (b) by equation (a)

$$\frac{Q_2}{Q_1} = \frac{\cancel{AR^{2/3}}/n_2}{\cancel{AR^{2/3}}/n_1} = \frac{n_1}{n_2}$$

$$Q_2 = \frac{n_1 Q_1}{n_2} = \frac{0.03}{0.02163} Q_1 = 1.387 Q_1$$

- ➢ The discharge increased by 38.7%

Example 2 A trapezoidal channel of bed width 3.00m and side slope 1.5horizontal to 1vertical carries a full supply of $10.00m^3$/sec. at a depth of 1.50m.

a) What would be the discharge at half of full supply depth (i.e at 0.75m)?
b) What would be the depth discharge at half of full supply discharge (i.e at $5.00m^3$/sec)?

Solution

- Wetted area (A) = (B+my)y =(3+1.5*1.5)*1.5 = $7.875m^2$
- Wetted Perimeter (P) = B+2y$\sqrt{1+m^2} = 3 + 2 * 1.5 * \sqrt{1 + 1.5^2} = 8.41m$
- Hydraulic Radius (R) = $\frac{A}{P} = \frac{7.875}{8.41} = 0.936m$
- Discharge (Q) = $\frac{AR^{2/3}\sqrt{S}}{n}$

❖ Hence $\frac{\sqrt{S}}{n} = \frac{Q}{AR^{2/3}} = \frac{10}{7.875 * 0.936^{2/3}} = 1.327$

a) Discharge at half of full supply depth (i.e at 0.75m)
   - Wetted Area (A) = (B+my)y = (3+1.5*0.75)*0.75 = $3.094m^2$
   - Wetted perimeter (P) = B+2y$\sqrt{1+m^2} = 3 + 2 * 0.75 * \sqrt{1 + 1.5^2} = 5.70m$
   - Hydraulic Radius (R) = $\frac{A}{P} = \frac{3.094}{5.70} = 0.54m$
   - Discharge (Q) = $\frac{AR^{2/3}\sqrt{S}}{n} = 3.094 * 0.54^{2/3} * 1.327 = 2.72\, m^3\ /\sec$

b) The depth discharge at half of full supply discharge (i.e at $5.00m^3$/sec)

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{5}{1.327} = 3.7679$$

With

$$A = (B + my)y = (3 + 1.5y)y$$

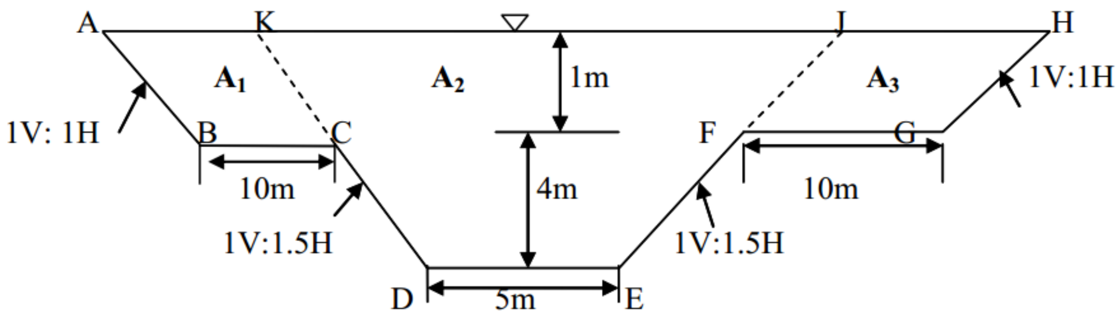
$$P = B + 2y\sqrt{1+m^2} = 3 + 2y\sqrt{3.25} = 3 + 3.6y$$

$$R = \frac{(3+1.5y)y}{3+3.6y}$$

Then the following table in constructed by assuming values of y and computing corresponding values of $AR^{2/3}$ matches the value computed from the problem statement, the corrected value of y_n has been determined as 1.043m

	Trial y, m	A, m^2	P m	R, m	$AR^{2/3}$	
	0.90	3.915	6.244996	0.626902	2.867685	
	0.95	4.20375	6.425274	0.654252	3.168111	
	1.00	4.5	6.605551	0.681245	3.484027	
	1.043	4.760774	6.76059	0.704195	**3.768248**	
	1.09	5.070974	6.940868	0.730596	4.113483	

Example: For the compound section shown in figure below. Find the discharge when h = 1 m. Assume n = 0.02 and S_o =0.0005 for all parts of the perimeter. Use Posey's method for computing partial discharges.



Solution:

Partial Area discharge by Posey's method:

Sub area A1 = 10*1+0.5*1*1-1.5*.5*1 =9.75m^2

Sub area 1 and sub area 3 are equal because they have the same dimensions =9.75m^2

Perimeter $P_1 = y\sqrt{1+m^2} + b = 1\sqrt{1+1^2} + 10 = 11.414m$ which is equal with P_3=11.414m

Sub area A_2 = (b+ m y) y = (5+1.5*5)5 =62.5m^2

Perimeter $P_2 = 2y\sqrt{1+m^2} + b = 2*5\sqrt{1+1.5^2} + 5 = 23.028m$

$$Qp_1 = \frac{A_1 R_1^{\frac{2}{3}} S_o^{\frac{1}{2}}}{n} = \frac{9.75*\left(\frac{9.75}{11.414}\right)^{\frac{2}{3}} * 0.0005^{\frac{1}{2}}}{0.02} = 9.813m^3/se$$

$$Qp_3 = \frac{A_3 R_3^{\frac{2}{3}} S_o^{\frac{1}{2}}}{n} = \frac{9.75*\left(\frac{9.75}{11.414}\right)^{\frac{2}{3}} * 0.0005^{\frac{1}{2}}}{0.02} = 9.813m^3/se$$

$$Qp_2 = \frac{A_2 R_2^{\frac{2}{3}} S_o^{\frac{1}{2}}}{n} = \frac{62.5 * \left(\frac{62.5}{23.028}\right)^{\frac{2}{3}} * 0.0005^{\frac{1}{2}}}{0.02} = 135.962 m^3 / se$$

$$Qp = \sum(Qp_1 + Qp_2 + Qp_3)$$

Qp =135.962+9.813+9.813

=155.588m^3/se

By the total-section Method (Using whole discharge method)

A_{total} =A=A_1+A_2+A_3 =9.75+9.75+62.5 = 82m^2

P_{total} = p = $1\sqrt{1+1^2} + 10 + 4\sqrt{1+1.5^2} + 5 + 4\sqrt{1+1.5^2} + 10 + 1\sqrt{1+1^2} = 42.251 m$

$$Qw = \frac{A R^{\frac{2}{3}} S_o^{\frac{1}{2}}}{n} = \frac{82 * \left(\frac{82}{42.251}\right)^{\frac{2}{3}} * 0.0005^{\frac{1}{2}}}{0.02} = 142.644 m^3 / se$$

Since QW <Qp the discharge in the channel is taken as Q = Qp =155.588m^3/sec.